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Steady-state conduction in stagnant beds of solid particles

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Abstract—A model is developed for predicting the influence of the containing walls on heat transfer processes in stagnant beds of solid particles. The model describes the effect of the walls on the mean voidage of the bed and on the mean effective thermal conductivity of the bed. The theoretical predictions are compared with available experimental data.

INTRODUCTION

STAGNANT fixed beds of solid particles are used in many areas of chemical reactor technology. In order to control the various chemical reactions the heat fluxes and the temperatures in the bed must be carefully controlled. Thus the modelling of the various heat transfer processes in these situations has received considerable attention [1].

In many situations it is possible to model the bed macroscopically by assuming that the bed consists of a homogeneous (or pseudohomogeneous) emulsion of solid particles and stagnant interstitial fluid. It is then assumed that the emulsion has constant and uniform voidage and that effective thermal conductivities can be assigned to it. Effective thermal conductivities of such systems have been extensively investigated, mainly by Kunii and Smith [2] and by Bauer and Schlünder [3].

However, the assumption of constant voidage emulsion does not hold in the vicinity of the walls of the bed. The presence of the walls disturbs the local packing and increases the voidage there. The influence of the increased voidage on the heat transfer properties of the bed depends on the thermal properties of the solid particles and the interstitial fluid. Usually this introduces an additional resistance to heat transfer, which may have an important influence on the overall heat transfer characteristics of the bed. This has been long recognized and, e.g. Yagi and Kunii [4] considered the additional surface thermal resistance for the case of packed beds and Baskakov and coworkers [5, 6] considered the resistance for the case of fluidized beds.

It should be noted that even though the influence of the wall on particle packing may propagate some distance from the wall (and thus for narrow beds may be experienced by the whole bed), the effect of the wall is mainly limited to a distance of about one particle diameter from the wall [7, 8]. Thus it seems reasonable to assume that the variation of the voidage is limited to the distance of one particle diameter from the wall and that then the voidage remains constant and undisturbed by the presence of the wall. Such variation of the voidage was used directly to develop a model of heat transfer in gas fluidized beds and the results of the model are in good agreement with the available experimental data [9, 10].

It is the purpose of this work to investigate the influence of the walls on steady-state conduction heat transfer in stagnant beds of solid particles by considering directly the effect of the walls on the variation of the voidage. First, the influence of the walls on the mean bed voidage is considered and the relationship between the mean voidage of the bed and the voidage of the undisturbed emulsion phase (i.e. undisturbed by the presence of the containing walls) is derived. Secondly, the influence of the walls on the mean effective thermal conductivity of the bed is analysed and the relationship between the mean effective thermal conductivity of the bed and the effective thermal conductivity of the undisturbed emulsion phase is derived. Finally, the theoretical results are compared with the available experimental evidence (mainly with the recent data of Melanson and Dixon [1]).

EFFECT OF THE WALLS ON MEAN BED VOIDAGE

One-dimensional (slab) bed

Consider a slab bed, such as that shown in Fig. 1. Since the width of the bed w is much smaller than either the height or the depth of the bed, it is assumed that the voidage of the emulsion varies only along the width of the bed. Let $\varepsilon(x)$ be the voidage of the emulsion phase anywhere in the bed. The mean voidage of the bed $\bar{\varepsilon}$ is then given as

$$\bar{\varepsilon} = \frac{1}{w}\int_0^w \varepsilon(x)\,\mathrm{d}x. \quad (1)$$

NOMENCLATURE

d	mean particle diameter
e	particle emissivity
h_W	apparent wall heat transfer coefficient
k_E	thermal conductivity of undisturbed emulsion phase
k_F	thermal conductivity of the fluid
k_P	thermal conductivity of the particles
$k(r)$	thermal conductivity of emulsion at r
$k(x)$	thermal conductivity of emulsion at x
$\bar{k}$	mean effective thermal conductivity of the bed
q	lateral heat flux
r	radius
R^c	contact resistance
T	temperature
T_E	emulsion phase temperature
T'	apparent wall temperature (see Fig. 1)
ΔT	temperature difference, $T_2 - T_1$
w	bed width
x	coordinate.

Greek symbols

ε_E	voidage of undisturbed emulsion phase
$\varepsilon(r)$	voidage of emulsion at r
$\varepsilon(x)$	voidage of emulsion at x
$\bar{\varepsilon}$	mean voidage of the bed.

Subscripts

i	inside wall
o	outside wall
1	wall 1 (see Fig. 1)
2	wall 2 (see Fig. 1).

Next it is assumed (as discussed above) that the voidage $\varepsilon(x)$ varies only up to the distance of one particle diameter from the containing wall and then that it remains constant and equal to the voidage of the undisturbed emulsion phase ε_E.

Hence equation (1) can be re-written as

$$\bar{\varepsilon} = \varepsilon_E + \frac{2}{w}\left[\int_0^d \varepsilon(x)\,dx - \varepsilon_E d\right] \tag{2}$$

where d is the mean particle diameter.

It is shown in ref. [9] that in fluidized beds the voidage variation in the vicinity of a flat wall can be approximated as

$$\varepsilon(x) = 1 - 3(1-\varepsilon_E)\left[\frac{x}{d} - \frac{2}{3}\left(\frac{x}{d}\right)^2\right]. \tag{3}$$

A more complex and detailed expression could be used for the case of stagnant beds, but equation (3) is assumed adequate for the approximate analysis presented in this work.

Equation (3) can then be substituted in equation (2) to obtain

$$\bar{\varepsilon} = \varepsilon_E + \frac{d}{3w}(1-\varepsilon_E). \tag{4}$$

It can be seen from equation (4) that the mean voidage of the bed is always greater than the voidage of the undisturbed emulsion phase, but that it approaches ε_E as the ratio w/d increases.

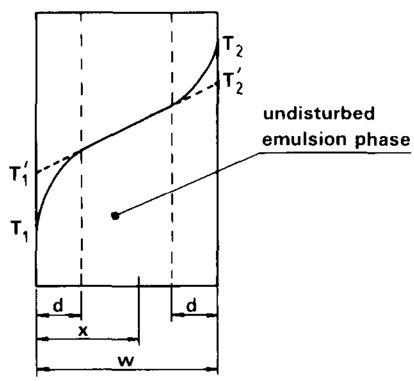

FIG. 1. A diagram of one-dimensional (slab) bed.

Annular bed

Consider an annular bed, such as that shown in Fig. 2, whose height is much greater than the outer radius of the bed r_o. The mean voidage of the bed is then given as

$$\bar{\varepsilon} = \frac{2}{r_o^2 - r_i^2}\int_{r_i}^{r_o} r\varepsilon(r)\,dr. \tag{5}$$

As discussed in ref. [10] the curvature of the containing surface may have some influence on the variation of the voidage in its vicinity. However, the expressions are quite (and unnecessarily) complex and their use is not warranted in the approximate analysis discussed in this work. Hence it is assumed that the voidage variation in the vicinity of either concave or convex surfaces is the same as the voidage variation

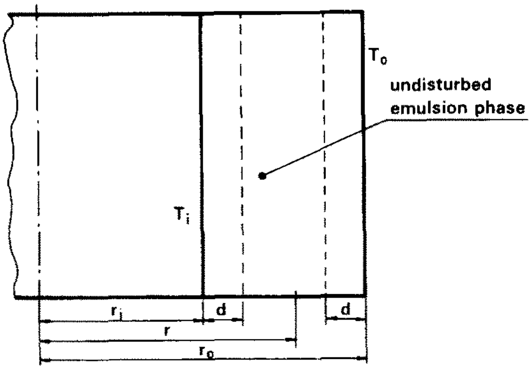

FIG. 2. A diagram of an annular bed.

in the vicinity of a flat surface, i.e.

$$\varepsilon(r) \equiv \varepsilon(x), \quad \text{with}\, x = r - r_i \quad \text{or} \quad x = r_o - r. \tag{6}$$

It can then be shown that

$$\bar{\varepsilon} = \varepsilon_E + \frac{d}{3(r_o - r_i)}(1 - \varepsilon_E). \tag{7}$$

Comparison with experimental data

Equations (4) and (7) can be best tested for relatively large ratios d/w or $d/(r_o - r_i)$ when the size of the solid particles is comparable to the 'width' of the bed. However, it should be noted that because of the various assumptions used in deriving equations (4) and (7), these equations are valid only for $w/d > 2$ and $(r_o - r_i)/d > 2$, respectively. Obviously the theoretical results can be extrapolated towards $w/d = 1$ and $(r_o - r_i)/d = 1$, but any agreement with the experimental data in that range must be regarded as fortuitous.

Three sets of experimental data for the mean voidage $\bar{\varepsilon}$ of beds of solid particles (i.e. particles without hollow spaces) have been examined: (i) experimental results of Melanson and Dixon [1] for annular beds of solid spheres and solid cylinders, (ii) experimental results of Yagi and Kunii [4] for annular beds of solid spheres and (iii) experimental results of Ofuchi and Kunii [11] for slab beds of solid spheres.

First, equation (7) was used to calculate the voidage of the undisturbed emulsion phase ε_E from the experimental data of Melanson and Dixon [1]. The mean value of ε_E was calculated as 0.346 with a standard deviation of 0.009. Secondly, equation (7) was also used to calculate ε_E from the experimental data of Yagi and Kunii [4]. The mean value of ε_E was calculated as 0.342 with a standard deviation of 0.021. Thirdly, equation (4) was used to calculate ε_E from the experimental data of Ofuchi and Kunii [11]. The mean value of ε_E was calculated as 0.345 with a standard deviation of 0.028.

Finally, the mean voidage $\bar{\varepsilon}$ was plotted as either the function of the ratio d/w (for slab beds) or the ratio $d/(r_o - r_i)$ (for annular beds) in Fig. 3. Figure 3

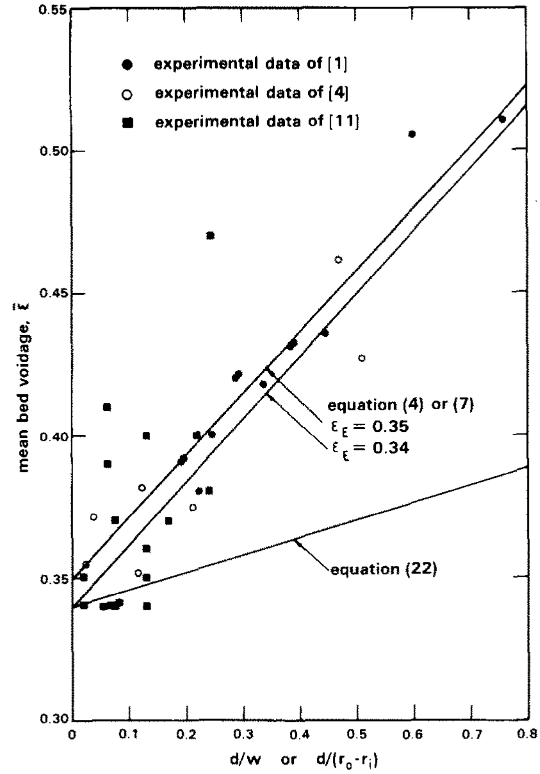

FIG. 3. A plot of the mean bed voidage $\bar{\varepsilon}$ against the ratio d/w (for slab beds) or $d/(r_o - r_i)$ (for annular beds).

also shows the theoretical results of equations (4) and (7) for two different values of the voidage of the undisturbed emulsion phase, $\varepsilon_E = 0.34$ and 0.35.

The comparison of the model with the experimental results thus shows that the voidage of the undisturbed emulsion phase is about 0.35 and that this is a good approximation even for beds with comparatively high values of particle diameter/bed width ratios. This value of the voidage of the undisturbed emulsion phase, i.e. $\varepsilon_E = 0.35$, will be used in the subsequent analysis.

EFFECTS OF THE WALLS ON BED THERMAL PROPERTIES

One-dimensional (slab) bed

As shown in Fig. 1, the two walls of the bed are held at constant temperatures T_1 and T_2, respectively. The governing equation for steady-state heat transfer by conduction is

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[k(x)\frac{\mathrm{d}T}{\mathrm{d}x}\right] = 0 \tag{8}$$

where $k(x)$ is the thermal conductivity of the emulsion anywhere in the bed.

First, we obtain the exact solution by taking into account the variation of $k(x)$ with x. The lateral heat flux across the bed is obtained from equation (8) as

$$q = \Delta T \Big/ \int_0^w \frac{dx}{k(x)} \tag{9}$$

where

$$\Delta T = T_2 - T_1. \tag{10}$$

Next it is assumed that the variation of the thermal conductivity of the emulsion phase is due only to the variation of the voidage of the emulsion phase. Since, as discussed above, the voidage is assumed to vary only within the distance of one particle diameter from the containing walls, the variation of the thermal conductivity is also limited to this distance and the thermal conductivity then remains constant and equal to the thermal conductivity of undisturbed emulsion phase. Using the above assumption equation (9) can be re-written as

$$q = \Delta T \Big/ \left\{ \frac{d}{k_E}\left[\frac{w}{d} + \frac{1}{d}\int_0^d \frac{k_E}{k(x)}dx - 1 + \frac{1}{d}\int_{w-d}^{w} \frac{k_E}{k(x)}dx - 1\right]\right\}. \tag{11}$$

Secondly, we assume that the bed can be associated with constant mean effective thermal conductivity $\bar{k}$. Subject to the constant surface temperature boundary condition, equation (8) can be solved as

$$q = \Delta T \Big/ \frac{w}{\bar{k}}. \tag{12}$$

Finally, we use the third model which idealizes the situation by assuming that the additional resistance to heat transfer in the vicinity of the heat transfer surface is concentrated on the heat transfer surface itself by providing additional contact resistance, and that the thermal conductivity anywhere in the bed (even in the vicinity of the heat transfer surface) is equal to the thermal conductivity of undisturbed emulsion phase k_E. The boundary conditions for this case thus become

$$\left.\begin{aligned} &\text{for } x = 0 \quad \frac{T_1 - T}{R_1^c} = -k_E\frac{dT}{dx} \\ &\text{for } x = w \quad \frac{T - T_2}{R_2^c} = -k_E\frac{dT}{dx} \end{aligned}\right\} \tag{13}$$

and the solution of equation (8) can be obtained as

$$q = \Delta T \Big/ \left[R_1^c + R_2^c + \frac{w}{k_E}\right]. \tag{14}$$

The relationship between the various heat transfer parameters can be obtained by comparing equations (11), (12) and (14) as

$$\frac{k_E}{\bar{k}} = 1 + \frac{d}{w}\left(\frac{k_E R_1^c}{d} + \frac{k_E R_2^c}{d}\right) \tag{15}$$

$$\frac{k_E R_1^c}{d} = \frac{1}{d}\int_0^d \frac{k_E}{k(x)}dx - 1 \tag{16}$$

$$\frac{k_E R_2^c}{d} = \frac{1}{d}\int_{w-d}^{w} \frac{k_E}{k(x)}dx - 1. \tag{17}$$

Annular bed

Using arguments similar to those above, the important heat transfer parameters for annular beds can be related as follows:

$$\frac{k_E}{\bar{k}} = 1 + \frac{1}{\ln r_o/r_i}\left[\frac{k_E R_i^c}{r_i} + \frac{k_E R_o^c}{r_o}\right] \tag{18}$$

$$\frac{k_E R_i^c}{r_i} = \int_{r_i}^{r_i + d} \frac{k_E}{r k(r)}dr + \ln\frac{r_i}{r_i + d} \tag{19}$$

$$\frac{k_E R_o^c}{r_o} = \int_{r_o - d}^{r_o} \frac{k_E}{r k(r)}dr + \ln\frac{r_o - d}{r_o}. \tag{20}$$

Heat transfer parameters

The various heat transfer parameters are related by equations (15)–(17) for slab beds and by equations (18)–(20) for annular beds. Equations (15) and (18) relate the mean bed effective thermal conductivity $\bar{k}$ with the effective thermal conductivity of undisturbed emulsion phase k_E. It can be shown that the ratio $k_E/\bar{k}$ is always greater than unity but that it approaches unity as the size of the bed increases compared with the particle diameter d.

It is not the object of this paper to investigate and predict k_E, since that, as discussed above, has been done extensively by other investigators [2, 3]. They give methods for relating k_E to the various parameters of the emulsion phase, such as k_P, k_F, ε_E, d, e and T_E. The first three parameters determine the contribution to heat transfer by conduction, whereas the second three parameters determine the contribution to heat transfer by radiation.

In order to calculate the heat transfer parameters of equations (15)–(20) the variation of thermal conductivity near the heat transfer surface must be known. This, as discussed above, depends on the variation of the emulsion phase voidage there. In this work the voidage variation given by equation (3) is used. The major reason for using this approach is that it was used successfully to describe heat transfer in gas fluidized beds [9]. The voidage variation of equation (3) is then used with the methods of Kunii and Smith [2] and Bauer and Schlünder [3] to calculate the variation of the thermal conductivity in the vicinity of the heat transfer surface.

COMPARISON WITH EXPERIMENTAL DATA

Initial observations

As shown by the previous experimental work [1, 4] the only thermal parameter which can be determined directly is the mean effective thermal conductivity of the bed, $\bar{k}$. This is determined from the heat flux and the mean temperature gradient over the whole bed (e.g. from $(T_2 - T_1)/w$ in the case of a slab bed). The thermal conductivity of the undisturbed emulsion phase k_E is obtained from the heat flux and the smoothed temperature gradient in the core of the bed, since it is assumed that the emulsion phase there is undisturbed by the containing walls.

The contact resistances are then determined from the heat flux and the difference between the wall temperatures and the temperatures which would be obtained on the walls if the temperature gradient in the core of the bed were extrapolated to the walls (e.g. from temperature differences $T_1' - T_1$ and $T_2 - T_2'$ in Fig. 1). This aspect is discussed more fully in refs. [1, 4].

Hence the mean bed thermal conductivity $\bar{k}$ is measured with the highest experimental accuracy, the experimental accuracy of k_E is lower and the experimental accuracy of the contact resistance R^c is lower still.

The experimental accuracy of measuring the contact resistance depends mainly on the accuracy with which we can determine the relevant temperature difference (such as $T_1' - T_1$ in Fig. 1). Since, for example, the temperature T_1' is determined by extrapolating the core temperature gradient, the accuracy of the temperature difference $T_1' - T_1$ depends directly on the magnitude of this temperature difference. Thus higher credence must be given to those experimental results which are based on higher values of the appropriate temperature difference.

It can be shown that, based on the above principle, the experimental data for the contact resistance on the outside walls of annular beds are much less reliable than those on the inside walls. This can be confirmed, e.g. by examining the consistency of the experimental data in ref. [1]. Furthermore, it can be also shown that the accuracy of the experimental data for the contact resistance decreases as the ratios $k_E R_1^c/w$, $k_E R_2^c/w$ or $k_E R_i^c/r_i$ decrease.

It should be noted that the theoretical results which are to be compared with the available experimental data must be calculated consistently. The techniques of Kunii and Smith [2] and of Bauer and Schlünder [3] can be used to calculate the thermal conductivity of the emulsion phase with constant and uniform voidage. Since the local voidage of the emulsion phase is also assigned a unique point value, it may be argued that the local value of the voidage of the emulsion phase can be used to calculate the local value of the thermal conductivity of the emulsion phase. Thus the only values of thermal conductivity which can be calculated directly are either the local thermal conductivity (which depends on the local voidage of the emulsion phase) or the thermal conductivity of the undisturbed emulsion phase (which depends on the uniform voidage of the undisturbed emulsion phase).

Hence, since the voidage of the emulsion phase varies over the bed (and is thus not uniform) it seems inappropriate to use the mean voidage of the bed to predict the mean thermal conductivity of the bed. The mean thermal conductivity of the bed can only be predicted indirectly from the core thermal conductivity by using, e.g. equations (15) and (18). Thus, there appears to be some inconsistency in the work of Melanson and Dixon [1] who used the mean bed voidage to calculate the mean bed thermal conductivity.

In order to use the two techniques [2, 3] for calculating the thermal conductivity of the emulsion phase the following bed parameters must be defined: the particle diameter d, the voidage of the undisturbed emulsion phase ε_E, particle emissivity e and the temperature of the bed.

The mean particle diameter is defined as a diameter of a sphere of the same volume as the volume of the particle based on its outside dimensions (i.e. any hollow spaces within the particle are neglected).

As indicated by Fig. 3 the voidage of the undisturbed emulsion phase of solid particles (i.e. particles without hollow spaces) is about 0.35, and this is the value used throughout this work. This value is also used for hollow particles since it is assumed that the internal voidage is of secondary importance because the heat conduction path round the hollow spaces is unbroken. Thus in applying the correlation of Kunii and Smith [2] $\varepsilon_E = 0.35$ was used, but in applying the corelation of Bauer and Schlünder [3] the required correction for the particle shape was taken into account as well.

Since the particle emissivities were not known they were inferred from the work of Melanson and Dixon [1] and judged for the remaining experimental work [4, 11]. The value of $e = 0.4$ was used for all the particles in this work with the exception of aluminium particles, for which $e = 0.1$ was used, and nylon and ceramic particles, for which $e = 0.5$ was used. Furthermore, since in all the reported experimental work the temperature of the bed generally increased from about 30°C to about 100°C, it is assumed that the bed is at a uniform temperature of 340 K.

Finally, since it is assumed that the bed temperature is uniform the variations of thermal conductivity of the emulsion phase in the vicinity of both heat transfer surfaces in a given bed are identical. Thus, for a one-dimensional bed equations (16) and (17) show that $R_1^c = R_2^c$. Hence equation (15) can be simplified as

$$\frac{k_E}{\bar{k}} = 1 + 2\frac{d}{w}\frac{k_E R_1^c}{d}. \tag{21}$$

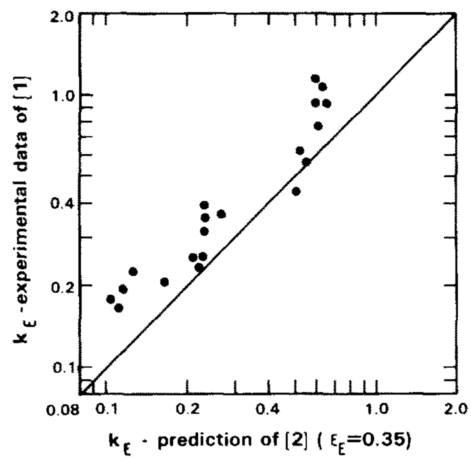

FIG. 4. A comparison of the experimental results of Melanson and Dixon [1] for k_E with the prediction of Kunii and Smith [2], using $\varepsilon_E = 0.35$.

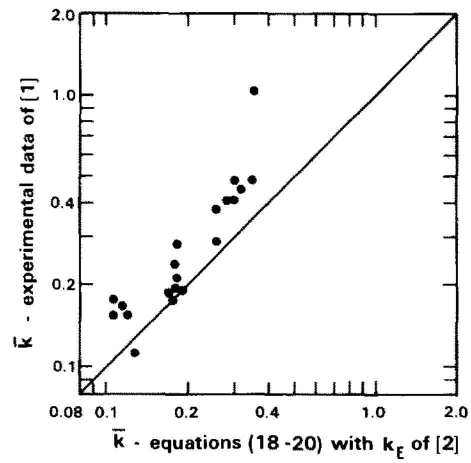

FIG. 6. A comparison of the experimental results of Melanson and Dixon [1] for $\bar{k}$ with the predictions of equations (18)–(20) and k_E based on the work of Kunii and Smith [2].

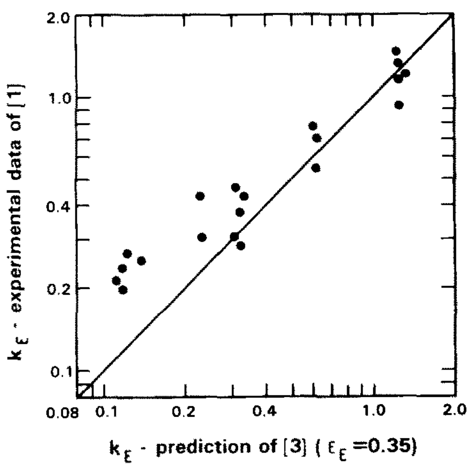

FIG. 5. A comparison of the experimental results of Melanson and Dixon [1] for k_E with the predictions of Bauer and Schlünder [3], using $\varepsilon_E = 0.35$.

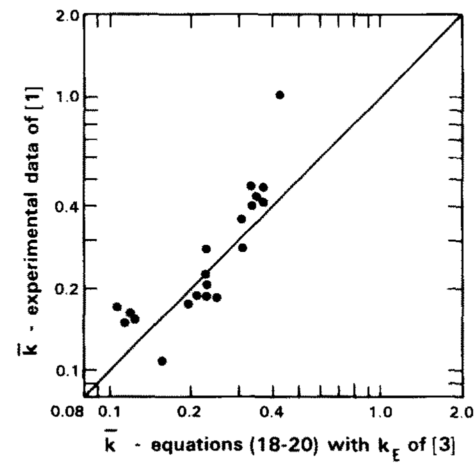

FIG. 7. A comparison of the experimental results of Melanson and Dixon [1] for $\bar{k}$ with the predictions of equations (18)–(20) and k_E based on the work of Bauer and Schlünder [3].

Thermal conductivity of undisturbed emulsion phase

As pointed out above, thermal conductivity of the emulsion phase has been extensively investigated by other authors. In this section experimental results of Melanson and Dixon [1] for the thermal conductivity of the undisturbed emulsion phase k_E will be compared with the formulas of refs. [2, 3] using the voidage of the undisturbed emulsion phase, $\varepsilon_E = 0.35$. As discussed above this is the voidage appropriate for calculating k_E. Figure 4 compares the experimental results with the formula of Kunii and Smith [2] and Fig. 5 with the formula of Bauer and Schlünder [3]. The two figures indicate that both methods appear to be underpredicting the experimental data but that the method of Kunii and Smith predicts the results more consistently. Thus the approach of Kunii and Smith [2] appears to be preferable. This agrees with the conclusion of Melanson and Dixon [1].

Mean effective thermal conductivity of the bed

The experimental results of Melanson and Dixon [1] for $\bar{k}$ in annular beds are compared with the theoretical results of equations (18)–(20) with k_E based on the work of Kunii and Smith [2] in Fig. 6 and with k_E based on the work of Bauer and Schlünder [3] in Fig. 7. Since the theoretical results are based on the available correlations of the thermal conductivity of the undisturbed emulsion phase k_E, which themselves do not compare particularly well with the experimental data of ref. [1], it is difficult to draw any conclusions from the above comparisons.

More insight may be obtained by examining the characteristics of the ratio $k_E/\bar{k}$. The experimental results of Melanson and Dixon [1] for the ratio $k_E/\bar{k}$ are compared with the theoretical results of equations (18)–(20) with k_E based on ref. [2] in Fig. 8 and with k_E based on ref. [3] in Fig. 9. The two figures indicate

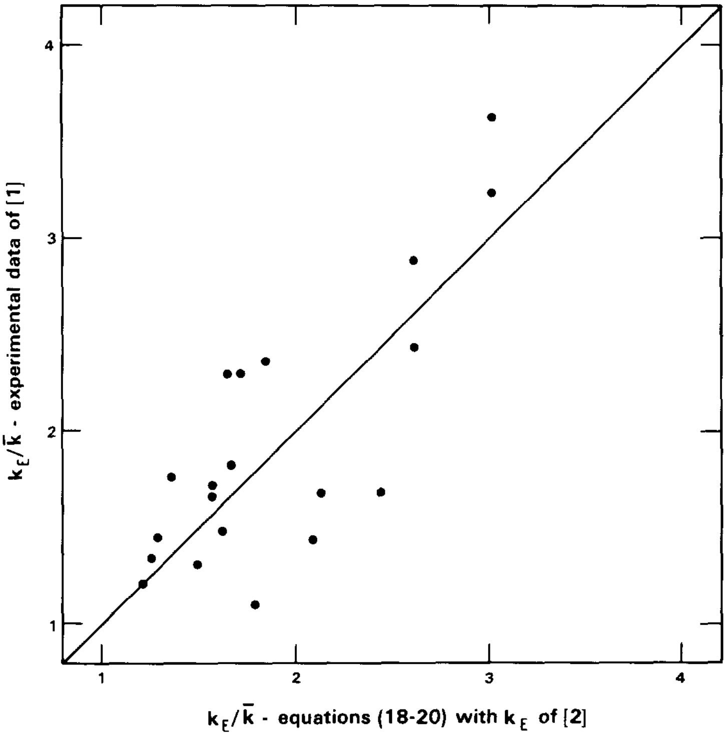

FIG. 8. A comparison of the experimental results of Melanson and Dixon [1] for $k_E/\bar{k}$ with the predictions of equations (18)–(20) and k_E based on the work of Kunii and Smith [2].

that, as in the previous section, the theoretical results based on the work of Kunii and Smith [2] give more consistent agreement with the experimental data.

Contact resistance

As discussed above the contact resistance is obtained from the heat flux and the temperature difference $T_1' - T_1$ or $T_2 - T_2'$. Since the measured temperature differences are generally small, large scatter in the data can be observed [1, 4, 11]. This must be kept in mind when comparing experimental results with the theoretical predictions. Furthermore, since the formula of Kunii and Smith [2] is preferable for predicting k_E, only their correlations are used in this section.

As pointed out above the experimental data for the contact resistance on the outside walls of annular beds are much less reliable than those on the inside walls. Thus for annular beds only those data of refs. [1,4] which refer to the inside wall are compared with the theoretical prediction of equation (19). The experimental results for $k_E R_i^c/r_i$ are compared with the theoretical predictions in Fig. 10. It should be noted that, as discussed above, the experimental data for small values of $k_E R_i^c/r_i$ are not very reliable. Removing those data for which $k_E R_i^c/r_i < 0.4$ improves the consistency of the comparison between experimental data and theoretical predictions. Figure 10 is replotted in Fig. 11 with $k_E R_i^c/d$ instead of $k_E R_i^c/r_i$, with those experimental data for which $k_E R_i^c/r_i < 0.4$ removed.

Experimental results of ref. [11] for $k_E R_1^c/d$ in a one-dimensional bed are compared with the theoretical predictions of equation (16) in Fig. 12. (It should be noted that only those experimental data which were based on the temperature difference $T_1' - T_1 > 2.5°C$ are included in the comparison.)

DISCUSSION

As pointed out above, in order to determine the thermal conductivity of the undisturbed emulsion phase k_E the voidage of the undisturbed emulsion phase ε_E must be used in the appropriate formulas. As indicated in Figs. 4 and 5 the two approaches for correlating k_E [2, 3] (and especially the more consistent method of Kunii and Smith [2]) underpredict k_E.

It is unlikely that the reason for this discrepancy is a systematic experimental error in the work of Melanson and Dixon [1], since (i) their experimental work was well controlled and (ii) the systematic error would have to be relatively high. Thus it is more likely that it is the correlating formulas which underpredict the experimental results. One parameter used in the correlating formulas which could be immediately questioned is the voidage ε_E, which was not determined directly but inferred (as discussed above). However, in order to reconcile theoretical predictions with the experimental data an emulsion voidage much lower than $\varepsilon_E = 0.35$ would have to be used. Since

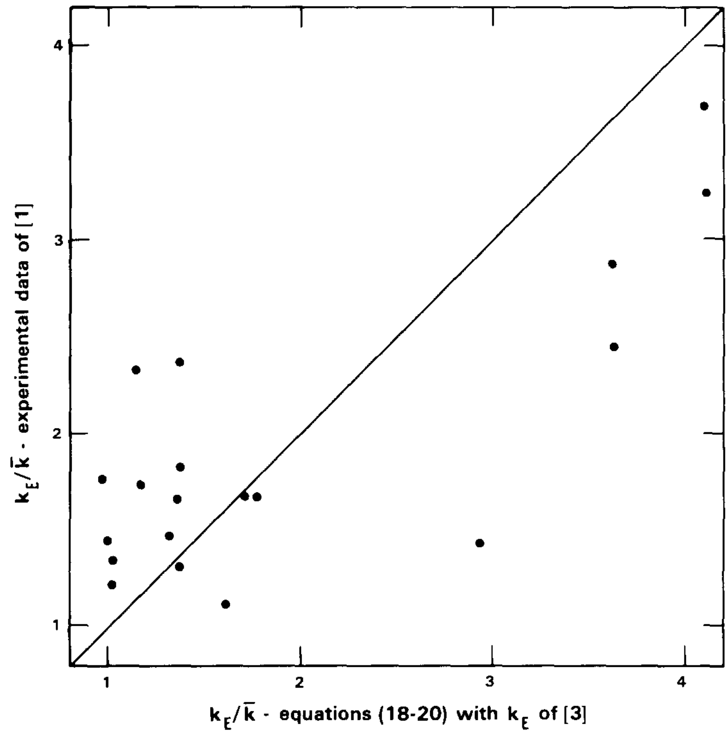

FIG. 9. A comparison of the experimental results of Melanson and Dixon [1] for $k_E/\bar{k}$ with the predictions of equations (18)–(20) and k_E based on the work of Bauer and Schlünder [3].

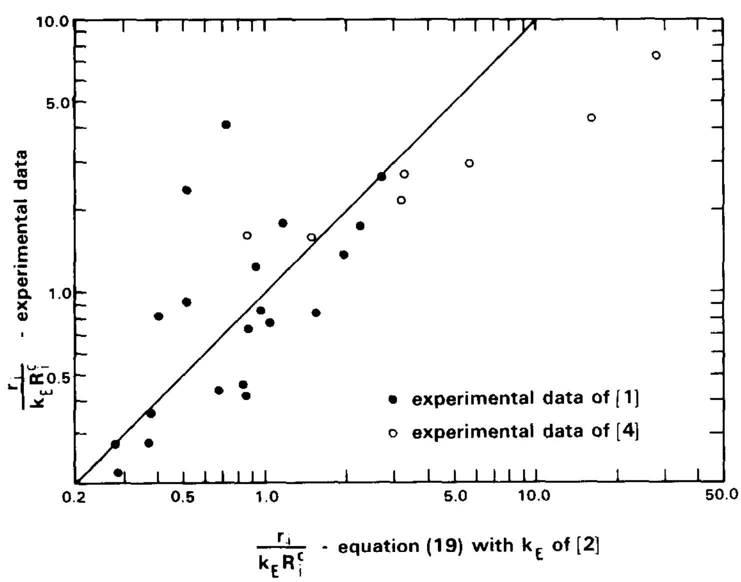

FIG. 10. A comparison of the experimental results of Melanson and Dixon [1] and Yagi and Kunii [4] for $k_E R_i^c/r_i$ with the theoretical predictions of equation (19) and k_E based on the work of Kunii and Smith [2].

the discussion in the above sections and Fig. 3 seem to show quite conclusively that the voidage ε_E is about 0.35, it seems unlikely that the voidage is much lower than 0.35. Hence it remains unclear why the correlating formulas underpredict the experimental results for k_E.

Similar trends can be observed for the theoretical predictions of $\bar{k}$, but as Fig. 8 indicates the ratio $k_E/\bar{k}$ is predicted quite consistently albeit with relatively large scatter. Since the theoretical predictions of $\bar{k}$ are based on the correlation formulas for k_E the reason for the underprediction of $\bar{k}$ is probably the same as the reason for the underprediction of k_E, and thus remains unclear. However, the fact that the ratio $k_E/\bar{k}$

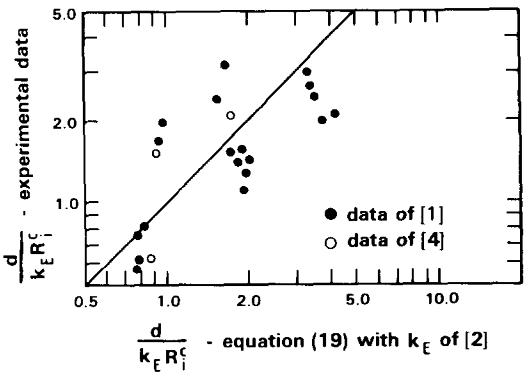

FIG. 11. A comparison of the experimental results of Melanson and Dixon [1] and Yagi and Kunii [4] for $k_E R_i^c/d$ with the theoretical predictions of equation (19) and k_E based on the work of Kunii and Smith [2].

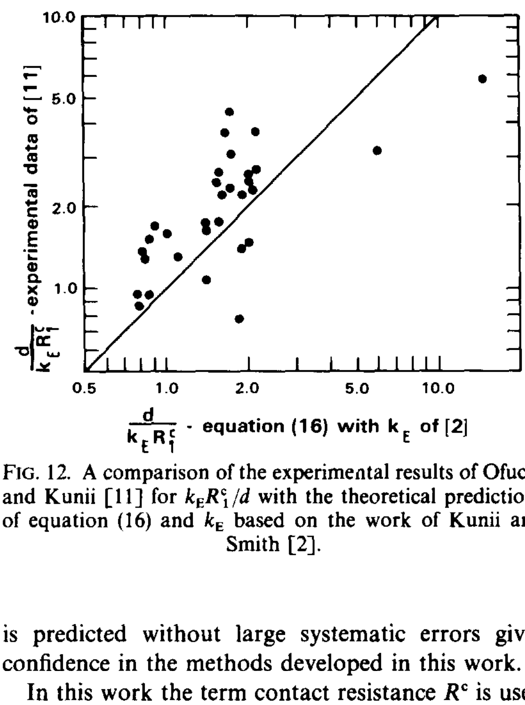

FIG. 12. A comparison of the experimental results of Ofuchi and Kunii [11] for $k_E R_1^c/d$ with the theoretical predictions of equation (16) and k_E based on the work of Kunii and Smith [2].

is predicted without large systematic errors gives confidence in the methods developed in this work.

In this work the term contact resistance R^c is used to describe the additional resistance to heat transfer near the heat transfer surfaces. This is related to its inverse $h_w = 1/R^c$, which is variously described as apparent wall heat transfer coefficient [1], apparent wall–film coefficient of heat transfer [11] or wall–film coefficient of heat transfer [4]. As can be observed [1, 4, 11] the experimental results for h_w in stagnant beds are subject to a large scatter. Thus it is not surprising that Figs. 10–12 also indicate a large scatter, but the figures show reasonable agreement between the theoretical predictions and the experimental results for contact resistance R^c.

Kunii and co-workers [11, 12] have developed a different model for h_w, based on two different values of the effective thermal conductivity of the emulsion phase: (i) thermal conductivity of the undisturbed emulsion phase k_E and (ii) thermal conductivity of the

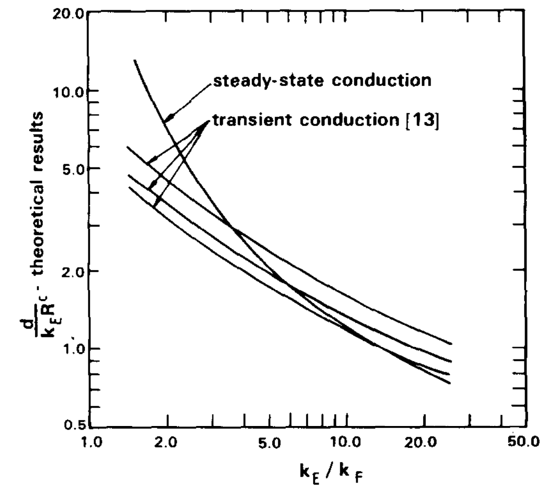

FIG. 13. A plot of $k_E R^c/d$ for flat heat transfer surfaces against k_E/k_F: a comparison of steady-state and transient conduction.

emulsion phase in the vicinity of the heat transfer surface. The latter conductivity is based on a rather artificial concept of the average voidage of the emulsion phase in the vicinity of the heat transfer surface, from the heat transfer surface to a distance of $d/2$. This voidage is assumed to be 0.40 and ε_E is assumed to be 0.34, which leads to the following equation for the mean voidage of the bed

$$\bar{\varepsilon} = 0.34 + 0.06\frac{d}{w}. \tag{22}$$

Equation (22) is also plotted in Fig. 3, which shows that the predictions of equation (22) diverge appreciably (and systematically) from the experimental data. Thus, even though the model of Kunii and co-workers [11, 12] for h_w (or R^c) gives as good a prediction as the model developed in this work, the present model which is based on more consistent assumptions is probably preferable.

If the bed particles are small or if the bed temperature is low the radiative component of heat transfer can be neglected, and the method of Kunni and Smith [2] correlates the thermal k_E with just k_P, k_F and ε_E. It can then be shown that the contact resistance is related to the bed properties by the following equation:

$$\frac{k_E R^c}{d} = \text{function of}\left\{\frac{k_E}{k_F}, \varepsilon_E, \text{surface geometry}\right\}. \tag{23}$$

For flat heat transfer surfaces the contact resistance was calculated from equation (16). The method of Kunii and Smith [2] was used and it was assumed that $\varepsilon_E = 0.35$. The theoretical results are presented in Fig. 13. This figure also shows contact resistances applicable to unsteady conductive heat transfer, which were obtained for various emulsion phase residence times in a study of heat transfer in gas fluidized beds

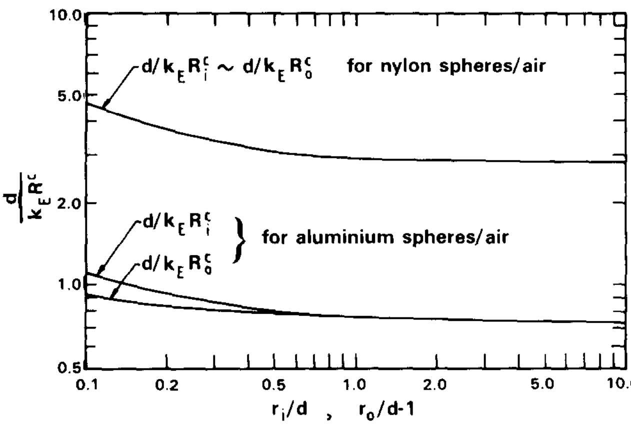

FIG. 14. The effect of the curvature of the heat transfer surface on contact resistance.

[13]. Since in the majority of practical applications $4 < k_E/k_F < 20$, Fig. 13 shows that the contact resistance obtained for steady-state conduction heat transfer can also be used to describe transient conduction heat transfer in packed or fluidized beds.

The effect of the curvature of the heat transfer surface on the contact resistance R^c is demonstrated in Fig. 14, which shows the plot of $d/k_E R_i^c$ and $d/k_E R_o^c$ vs r_i/d and r_o/d, respectively, for two different beds: (i) nylon spheres in air and (ii) aluminium spheres in air. The theoretical results were obtained from equations (19) and (20) using the method of Kunii and Smith [2] to calculate k_E and $k(r)$.

It should be noted that the present model is probably not valid for the cases when r_i/d approaches zero and r_o/d approaches unity. Nevertheless, the model indicates that the curvature of the heat transfer surface has only a small influence on the values of the contact resistance. For example, for $r_i/d > 0.5$ and $r_o/d > 1.5$ the contact resistances are within 10% of the contact resistance on a flat heat transfer surface. Hence it appears that the influence of the curvature of the heat transfer surface on the contact resistance can be neglected and that the values of contact resistance obtained on a flat heat transfer surface can be used instead.

The theoretical results of Fig. 14 are re-plotted on Fig. 15 as $r_i/k_E R_i^c$ and $r_o/k_E R_o^c$ vs r_i/d and r_o/d, respectively. It can be observed from Fig. 15 that whereas $r_i/k_E R_i^c$ decreases as r_i/d decreases, $r_o/k_E R_o^c$ reaches its minimum and then starts increasing with decreasing r_o/d. Melanson and Dixon [1] plotted their experimental data in this form and concluded that for $r_o/d < 2$ there is a strong upturn in $r_o/k_E R_o^c$ as r_o/d decreases. They supported this observation by arguing that for $r_o/d = 0.5$ there is only solid in the bed and thus that the contact resistance would tend to zero. This is incorrect since for $r_o/d = 0.5$ the voidage is 0.33. Hence it is difficult to decide whether or not there is a strong upturn in $r_o/k_E R_o^c$ as r_o/d decreases below 2. It should be pointed out that in this region the results are of not much practical interest, since for $r_o/d < 2$ the whole concept of constant core properties with surface contact resistance breaks down because the wall effects will propagate throughout the bed.

Finally, the heat transfer parameters were related in this model on the basis of equivalent heat flux, which is of course the dominant parameter in this application. Thus the model is only applicable to steady-state conduction heat transfer in slab or annular beds. For other bed geometries or other heat transfer processes the model must be derived from the first principle, as discussed above.

CONCLUSIONS

A model has been developed for predicting the influence of the containing walls on steady-state conduction heat transfer in stagnant beds of solid particles. Two bed geometries were considered: one-dimensional (slab) beds and annular beds.

The effect of the containing walls on the bed voidage was considered first, and the variation of the bed voidage due to the presence of the containing walls was used to relate the various heat transfer parameters, such as the contact resistance, the mean effective bed thermal conductivity and the thermal conductivity of the emulsion phase in the core of the bed.

The present model is based on the work of Kunii and Smith [2] on the thermal conductivity of the uniform emulsion phase, since it has been shown that their work gives more consistent predictions than the work of Bauer and Schlünder [3].

The theoretical predictions of the present model for the mean voidage of the bed, the mean effective thermal conductivity of the bed and the contact resistance are shown to be in reasonable agreement with available experimental data.

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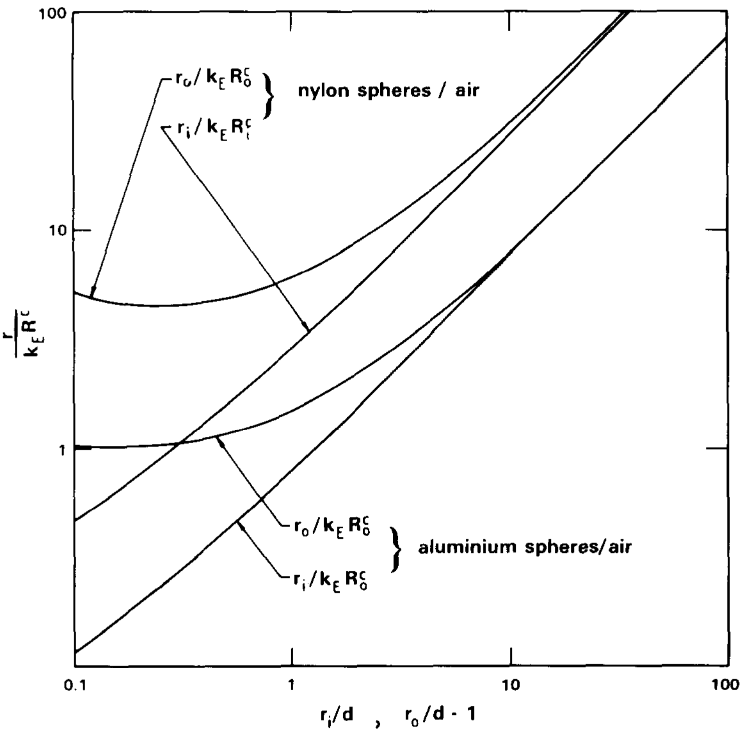

FIG. 15. A plot of $r_i/k_E R_i^c$ and $r_o/k_E R_o^c$ against r_i/d and r_o/d, respectively.

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CONDUCTION PERMANENTE DANS DES LITS FIXES DE PARTICULES SOLIDES

Résumé—On développe un modèle pour estimer l'influence des parois de l'enceinte sur les mécanismes de transfert thermique dans les lits fixes de particules solides. Le modèle décrit l'effet des parois sur la conductivité thermique effective moyenne du lit. Les prédictions théoriques sont comparées avec des données expérimentales disponibles.

STATIONÄRE WÄRMELEITUNG IN FESTBETTEN AUS FESTKÖRPERPARTIKELN

Zusammenfassung—Zur Vorhersage des Einflusses der Begrenzungswände auf den Wärmetransport in ruhenden Festbetten mit festen Partikeln wurde ein Modell entwickelt. Das Modell beschreibt den Einfluß der Wände auf die mittlere Porosität und die mittlere effektive Wärmeleitfähigkeit des Festbettes. Die theoretischen Vorhersagen werden mit verfügbaren experimentellen Daten verglichen.

СТАЦИОНАРНАЯ ТЕПЛОПРОВОДНОСТЬ В НЕПОДВИЖНЫХ СЛОЯХ ТВЕРДЫХ ЧАСТИЦ

Аннотация—Разработана модель для расчета влияния стенок на процессы теплопереноса в неподвижных слоях твердых частиц. Модель описывает влияние стенок на среднюю пористость и среднюю эффективную теплопроводность слоя. Теоретические результаты сравниваются с имеющимися экспериментальными данными.